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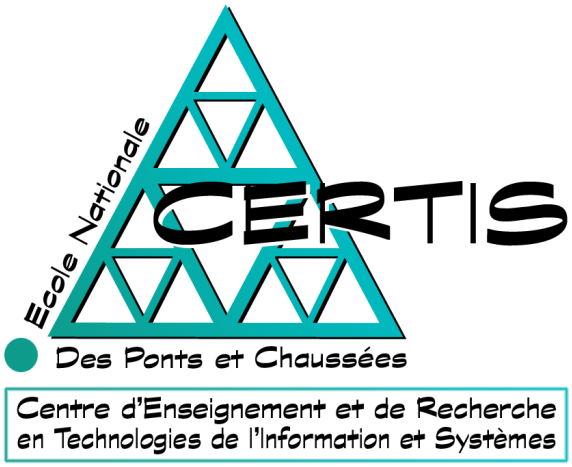

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Image Statistics based on Diffeomorphic Matching

Statistiques d'images basées sur la mise en correspondance via des difféomorphimes

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Abstract

We propose a new approach to deal with the first and second order statistics of a set of images. These statistics take into account the images characteristic deformations and their variations in intensity. The central algorithm is based on non-supervised diffeomorphic image matching (without landmarks or human intervention). Such statistics of sets of images may be relevant in the context of object recognition. The proposed approach has been tested on a small database of face images to compute a mean face and second order statistics. The results are encouraging. As a step further toward the evaluation of the approach, we present facial expression recognition experiments. We test the recognition of the facial expression of someone with and without the knowledge of his/her face with no expression.

Résumé

Nous proposons une nouvelle approche des statistiques de premier et second ordre d'un ensemble d'images. Ces statistiques prennent en compte les déformations caractéristiques des images, ainsi que les variations de leur intensité. L'algorithme central est basé sur la mise en correspondance non-supervisée d'images via des difféomorphismes (sans ajout manuel de points de correspondance ou autre intervention humaine). De telles statistiques d'ensembles d'images peuvent se montrer pertinentes dans le contexte de la reconnaissance d'objets. L'approche proposée a été testée sur une petite base de photographies de visages, et aboutit au calcul d'un visage moyen et de statistiques de second ordre. Les résultats sont encourageants. Afin d'évaluer l'approche, nous présentons des tâches de reconnaissance d'expressions, où nous testons la reconnaissance de l'expression faciale d'une personne avec ou sans la connaissance de son visage sans expression.

Contents

1. Introduction

How to find or recognize an object in an image? This is one of the most outstanding open problems in computer vision. Its solution will require a better understanding of the various possible visual aspects of a given object or a class of objects. For example, in the case of faces the description should include variations due to viewpoint, illumination, expression (happiness, surprise, ...), or the identity of the person. Like [3, 4] we think that statistics on images are necessary in order to tackle this problem. What we propose in this article is in a sense an extension to the set of images of an object of the work done on the statistics of 2D or 3D shapes [7, 1, 6]: by computing, from a set of images of a class of objects, the various ways these images can be warped onto one another we define and compute a mean image for that class and its second order statistics. Note that unlike previous approaches, e.g., [4] our approach does not require any manual intervention to identify landmarks or regions of interest. We work directly on the deformation fields which establish the correspondences between the whole images, since these fields are the fundamental elements of the problem. In order to do this we build upon previous work on non-supervised algorithms that build such correspondence fields between images, e.g., [7, 8, 5, 2]

In Section 2 we model the matching problem between two images and describe a variation of a matching algorithm proposed in [5] and analyzed in [2]. In Section 3 we use it to define and compute the mean image of a set of images and in Section 4 to define and compute its second order statistics. Then in Section 5 we show how to use the mean image in an expression recognition task.

2. Image matching

The main difficulty when defining the mean of several images is that this mean is supposed to *look like* each one of the images. This implies that the images have been registered and this is why we consider now the matching problem.

2.1. Basic framework

Let A and B be two images. We think of them as positive real functions functions defined in a rectangular subset Ω of the plane $\mathbb{R}^2$. We search for a deformation field $\mathbf{f}$ such that the warped image $A \circ \mathbf{f}$ resembles B. More precisely, we would like the field $\mathbf{f}$ to be smooth enough and invertible, i.e. it should be a diffeomorphism from the rectangular subset Ω to itself, which leads us to assume that the diffeomorphism $\mathbf{f}$ equals the identity on the image boundary $\partial\Omega$. Other possibilities are offered by extending the images to a larger subset Ω_1.

In order to keep $\mathbf{f}$ continuous, we have to consider a regularizing term $R(\mathbf{f})$

on $\mathbf{f}$, for example $R(\mathbf{f}) = \|\mathbf{f} - Id\|_\Omega^{H^1}$ where Id is the identity function on Ω and $\|a\|_\Omega^{H^1} = \int_{x\in\Omega} \|a(x)\|^2 + \|Da(x)\|^2\, dx$, or even, if we prefer to be sure $\mathbf{f}$ remains invertible, $\|\mathbf{f} - Id\|_\Omega^{H^1} + \|\mathbf{f}^{-1} - Id\|_\Omega^{H^1}$, where $\mathbf{f}^{-1}$ is the inverse of $\mathbf{f}$.

Now we have to choose a criterion $C(A, B)$ which expresses the similarity between the two images A and B. The simplest one is $C(A, B) = \|A - B\|_\Omega^{L^2} = \int_{x\in\Omega} (A(x) - B(x))^2\, dx$, but we prefer the following one, which has the advantage of being based on intensity variations and consequently the one of being contrast-invariant.

2.2. Local Cross-Correlation

Given a scale σ, the cross-correlation of two images A and B at point x is defined by:

$$CC(A, B, x) = \frac{v_{AB}(x)^2}{v_A(x)\; v_B(x)}$$

where $v_A(x)$ is the local spatial variance of A in a gaussian neighborhood of size σ centered on x, and $v_{AB}(x)$ the local covariance of A and B on the same neighborhood, i.e. we define:

$$g(x, y) = e^{\frac{\|x-y\|^2}{2\sigma^2}}$$

$$\mu(x) = \int_{y\in\Omega} g(x, y)\; dy$$

$$\bar{A}(x) = \frac{1}{\mu(x)} \int_{y\in\Omega} A(y)\; g(x, y)\; dy$$

$$v_A(x) = \epsilon + \frac{1}{\mu(x)} \int_{y\in\Omega} (A(y) - \bar{A}(x))^2\; g(x, y)\; dy$$

$$v_{AB}(x) = \frac{1}{\mu(x)} \int_{\Omega} (A(y) - \bar{A}(x))(B(y) - \bar{B}(x))\; g(x, y)\; dy$$

The positive constant ϵ is added only not to have a null divider in the expression of $CC(A, B, x)$. Given this, the local cross-correlation on the whole images are defined by [5]:

$$LCC(A, B) = \int_{x\in\Omega} CC(A, B, x)\; dx$$

2.3. The Image Matching Algorithm

The matching algorithm consists in minimizing with respect to the deformation field $\mathbf{f}$ (initialized to the identity) through a multi-scale gradient descent the following energy (see [2] for details)

$$E(A, B, \mathbf{f}) = LCC(A \circ \mathbf{f}, B) + R(\mathbf{f})$$

3. The mean of a set of images

Now that we know how to compute a diffeomorphic matching between two images, we can try to infer from this a new algorithm for the computation of the mean of n images A_i indexed by $i \in \{1, \ldots, n\}$. This is not as easy as one could guess.

3.1. An intuitive algorithm: find the mean

We can first define the mean as the image M which looks the most like all the warped images, i.e., if we introduce n diffeomorphisms $\mathbf{f}_i$ in order to warp an image A_i on the mean M, we could minimize

$$\sum_i E(A_i \circ \mathbf{f}_i, M, \mathbf{f}_i)$$

with respect to M and the fields $\mathbf{f}_i$. But how do we choose the initial image M? Besides, here is the main problem: we should not minimize the energy E with respect to an image. Indeed, if we consider the case where $n = 2$ and the two images are the same one translated by a few pixels, the gradient term due to the diffeomorphisms should move them so as to find the translation, but this is prevented by the minimization with respect to the mean image M, which, by averaging the intensities, introduces new contours induced by those in the two images. Consequently, each of the two images "sees" its contours appear in M at the same location, and the diffeomorphisms will not evolve from the identity.

3.2. Another intuitive algorithm

We can then try to substitute in E an expression for M as a function of the diffeomorphisms, effectively eliminating the unknown M. For example, we can choose $M = \frac{1}{n}\sum_i A_i \circ \mathbf{f}_i$ and minimize with respect to the $\mathbf{f}_i$ the following criterium:

$$\sum_i E(A_i \circ f_i, \frac{1}{n}\sum_k A_k \circ \mathbf{f}_k, \mathbf{f}_i)$$

We then encounter another problem: we try to match for each i $A_i \circ \mathbf{f}_i$ and $\frac{1}{n}\sum_i A_i \circ \mathbf{f}_i$; however, as $\frac{1}{n}\sum_i A_i \circ \mathbf{f}_i$ is the sum of the warped images, it contains in particular all the contours of $A_i \circ \mathbf{f}_i$, which means that we still have the same problem as before: the diffeomorphisms are immediatly stuck in a local minimum.

3.3. The final word: eliminating the mean

The problem comes mostly from the fact that we are trying to work directly on the mean of the images, whereas we should work only with the fields $\mathbf{f}_i$, which

carry all the information about the problem. Indeed, the mean M contains much less information than the diffeomorphisms $\mathbf{f}_i$: for example the mean of a white disk on a black background and a black disk on a white background is uniformly grey and consequently has not a large LCC-correlation with the initial images. Therefore we should rather deal with pairs of warped images than with pairs of a warped image and the mean. The mean then becomes an auxiliary quantity, just computed at the end when the diffeomorphisms are known.

The algorithm proceeds as follows: initialize all deformation fields $\mathbf{f}_i$ to the identity, and let them evolve in a multiscale framework in order to minimize

$$\frac{1}{n-1}\sum_{i\neq j} LCC(A_i \circ \mathbf{f}_i, A_j \circ \mathbf{f}_j) \quad + \quad \sum_k R(\mathbf{f}_k)$$

Thus, at the end of the evolution, each $A_i \circ \mathbf{f}_i$ is supposed to look like each of the others, and the mean is naturally computed as $M = \frac{1}{n}\sum_i A_i \circ \mathbf{f}_i$. The regularizing term $\sum_i R(\mathbf{f}_i)$ implies that if several sets of fields $\mathbf{f}_i$ conduct to approximatively the same energy $\sum_{i\neq j} LCC(A_i \circ \mathbf{f}_i, A_j \circ \mathbf{f}_j)$ (for example by adding a common diffeomorphism $\mathbf{f}_c$ to every field and replacing $\mathbf{f}_i$ with $\mathbf{f}_i \circ \mathbf{f}_c$), then the most intuitive one is chosen (the one of least regularizing cost). We also impose the condition $\sum_i \mathbf{f}_i = 0$ at each time step by substracting the mean of the fields $\frac{1}{n}\sum_i \mathbf{f}_i$ to each of them. This may be questionable but leads to good results in practice.

3.4. Example

We have tested this algorithm on a face database from Yale[2]. More precisely, we have computed the mean face out of photographs of ten different people with similar expressions, approximatively the same illumination and position conditions, and the same size (195 * 231 pixels). The ten image A_i are shown in figure 1, the ten warped images $A_i \circ \mathbf{f}_i$ in figure 2, and their mean in figure 3.

Note the accuracy of the mean: it looks like a real face, its features are not blurred at all (except the ears). The strange white curved line below the eyes comes from the reflects of the light into the eighth man's glasses. The computation takes about 10 minutes on a standard workstation. Also note the good job done by the diffeomorphisms $\mathbf{f}_i$, see figure 2.

4. Second order statistics of a set of images

As the information about the shape variations in the set of images A_i lies in the diffeomorphisms $\mathbf{h}_i$, we compute statistics on these warping fields.

[2] `http://cvc.yale.edu/projects/yalefaces/yalefaces.html`

Figure 1: The ten face images.

Figure 2: The ten warped images $A_i \circ \mathbf{f}_i$.

Figure 3: The mean of the previous ten faces.

4.1. Definition and computation

These fields are functions from a subset Ω of the plane $\mathbb{R}^2$ to itself, therefore the natural way to express correlation between two fields $\mathbf{a}$ and $\mathbf{b}$ is to compute their scalar product for the usual norm $L^2(\Omega \to \mathbb{R}^2)$:

$$\langle \mathbf{a} \,|\mathbf{b} \rangle_{L^2(\Omega \to \mathbb{R}^2)} = \frac{1}{|\Omega|} \int_\Omega \mathbf{a}(x) \cdot \mathbf{b}(x) \, dx$$

Since the mean $\bar{\mathbf{f}}$ of the fields $\mathbf{f}_i$ is 0 (see above), the (shape-)correlation matrix SCM defined by $SCM_{i,j} = \left\langle \mathbf{f}_i - \bar{\mathbf{f}} \,\middle|\, \mathbf{f}_j - \bar{\mathbf{f}} \right\rangle_{L^2(\Omega \to \mathbb{R}^2)}$ can be simplified in $SCM_{i,j} = \langle \mathbf{f}_i \,|\mathbf{f}_j \rangle_{L^2(\Omega \to \mathbb{R}^2)}$. Then we diagonalize the correlation matrix SCM (its size is the number of images, not the number of pixels), and extract its eigenvalues σ_k and normalized eigenvectors $\mathbf{v}_k$. The modes of deformation are then defined by $\mathbf{m}_k = \sum_i (\mathbf{v}_k)_i \, \mathbf{f}_i$.

As statistics were made in the linear space $L^2(\mathbb{R}^2 \to \mathbb{R}^2)$, we can continuously apply a mode $\mathbf{m}_k$ to the mean image M with an amplitude α ($\in \mathbb{R}$) by computing the image $M \circ (Id + \alpha(\mathbf{m}_k - Id))$, and then produce animations of the deformations.

4.2. Example

These modes are illustrated in figure 4. Each column represents a mode, starting from the main one (leftmost column) to the one with the smallest eigenvalue, which is actually 0 because of the constraint on the mean field (rightmost column). Each column is divided in five images: in the central image, we represent the mean we computed before; in the images just above and underneath the mean, we represent the mode applied to the mean with amplitude $+\sigma_k$ and σ_k; and then with amplitude $+2\sigma_k$ and $-2\sigma_k$ in first and last image of each column, in order to exaggerate and better visualize the deformations.

4.3. Intensity variations

In order to take all the face variations into account, we should not only consider the shape variations (i.e. the diffeomorphisms) but also the intensity variations. As before, we can define an intensity-correlation matrix ICM on the intensity variations $I_i = A_i \circ \mathbf{f}_i - M$ for the $L^2(\mathbb{R}^2 \to \mathbb{R})$ scalar product. Thus, we can compute the principal modes of intensity variations, which correspond to skin or hair changes for a shape-fixed head.

We can also combine shape and intensity variations. If we note $\sigma_S^2 = \frac{1}{n} \sum_i \|\mathbf{f}_i\|^2$ and $\sigma_I^2 = \frac{1}{n} \sum_i \|I_i\|^2$ the standard deviations of shapes and intensities, we can

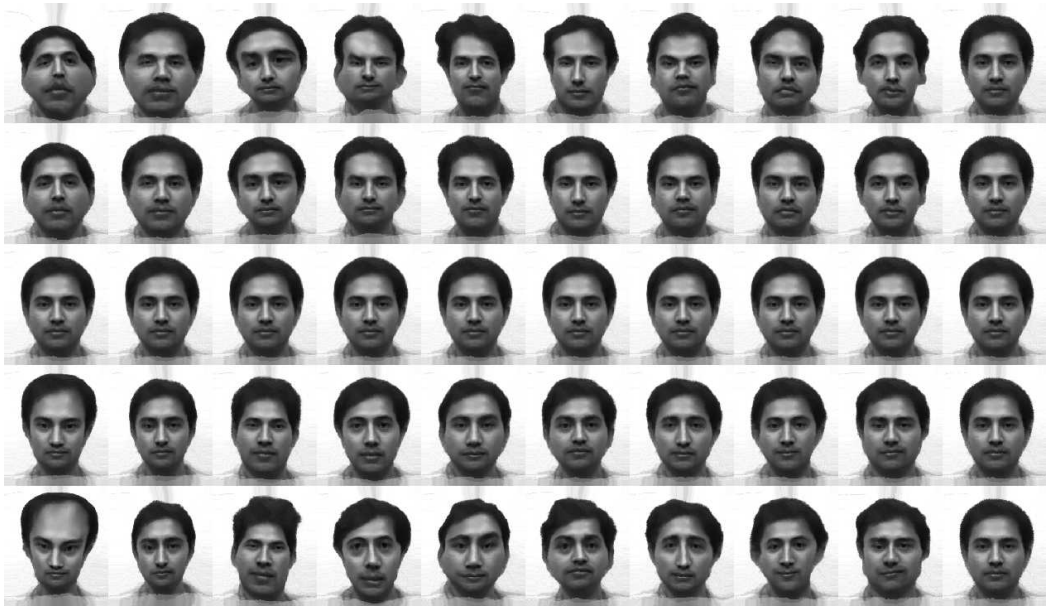

Figure 4: The shape modes of deformation of the previous set of images. Each column represents a mode, applied to the mean image with amplitude $\alpha = \{2\sigma_k, \sigma_k, 0, -\sigma_k, -2\sigma_k\}$.The (relative) values of the eigenvalues are, from left to right, 1, 0.5, 0.3, 0.1,..., 0.05, 0.

define a combined correlation matrix $CCM = 1/\sigma_S^2 SCM + 1/\sigma_I^2 ICM$ and proceed as before, compute and display principal modes of variations. The results are shown on figure 5. Note how these faces are realistic and diversified.

Figure 5: The eight non-zero combined modes of deformation of the same set of images without the subject with glasses (see footnote 3). Each column represents a mode, applied to their mean image with amplitude $\alpha = \{\sigma_k, -\sigma_k\}$. The (relative) values of the eigenvalues are, from left to right, 1, 0.555, 0.505, 0.424, 0.286, 0.232, 0.162, 0.135.

5. Classification: Expression Recognition

Let us now consider the facial expression recognition task. The goal is to associate with any new face its expression. We still use the Yale database. We remove from this database the 2 subjects wearing glasses[3] and we consider the 5 following facial expressions: happy, sad, sleepy, surprised and winking, beside the "normal" one.

5.1. From the mean image

The following simulations show that deformations from a mean face can be used to classify facial expressions. More precisely we choose as a reference face the mean "normal" face of the first 9 subjects of the database. Our first classification procedure uses a Support Vector Machine with Gaussian kernel[4] on the deformations from this face to expressive faces. To measure the efficiency of the method, we cross-validate the errors by taking out one subject among the 13 subjects in the database and consequently using 60 faces labeled by their expression to deduce the expression of the five remaining faces[5]. The cross-validation error is 24 upon 65 faces. For comparison purposes, we trained a Support Vector Machine with Gaussian kernel[6] using only the gray level intensity information. In this case we obtained a larger cross-validation error of 27 upon 65 faces which shows the interest of using the diffeomorphisms.

5.2. With knowledge of the face without expression

The advantage of using the deformations instead of the gray level intensities is even larger when we know whose face it is that we want to process. More precisely, if we use the subject's "normal" face to compute the deformation between the expressive one and classify this "expression" deformation after alignment on the mean face (using the "subject" deformation between the mean face and the face without expression), the cross validation error goes down to 12 (upon 65), whereas the classification using the difference of gray level intensities between the expressive face and the normal one leads to 17 errors.

These results, although preliminary, indicate that the mean image can be very useful in a classifying task, considering that the database is small, that the pro-

[3] Glasses introduces strong intensity gradient in the middle of the face. As a consequence, it is not relevant to try to deform a face without glasses to a face with glasses.

[4] The bandwidth of the kernel is equal to the median of the norms of the difference between 2 deformations of the training set.

[5] Thus we have no prior information on the subject to classify his facial expressions.

[6] The bandwidth of the kernel is the median of the norms of the intensity difference between 2 images of the training set.

cedures were not specialized to faces, and that even a human classifier may have problem with some of the considered faces (see figure (6) again)!

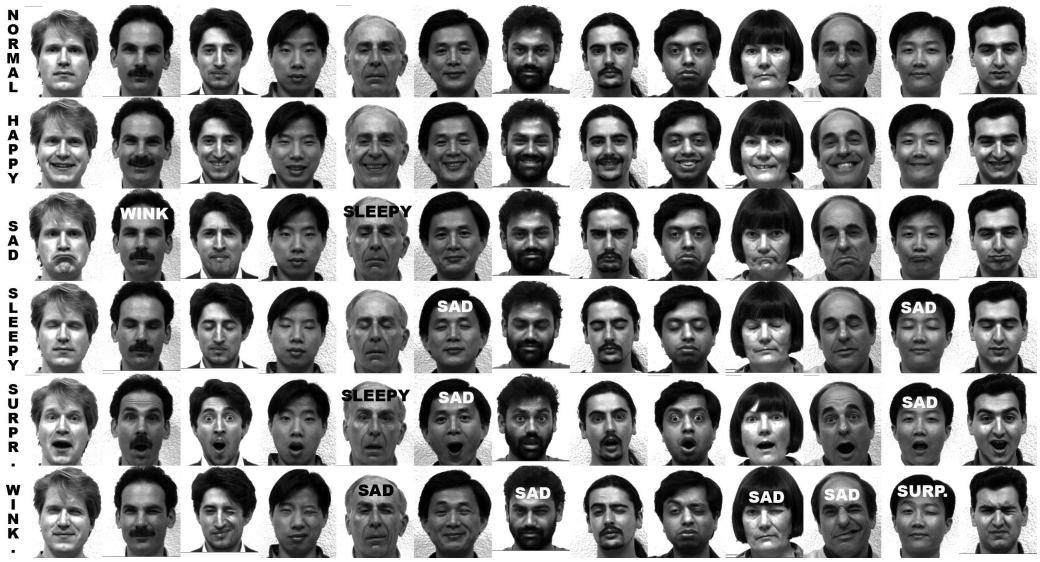

Figure 6: Expression recognition using deformations of the normal face, aligned with the mean face. From top to bottom: normal, happy, sad, sleepy, surprised and winking faces. 53 of the 65 expressive faces are correctly classified, 12 are not. For these we show the incorrect label.

6. Summary and Conclusions

We have defined and computed first and second order statistics of a set of images with a diffeomorphic matching approach (without landmarks), and shown how to use them in a classification task. We have tested this general approach on a face database, and the results are encouraging: the mean face looks like that of a real human being, the modes of variations are very sensible, and the expression recognition results are good, especially if we are also given a ”normal” image of the face to classify. We insist on the fact that our methods are not specific to faces and do not use any prior on the kind of images. We are in the process of including the second order statistics in the classification algorithm.

Acknowledgments

The SVM were run using the Spider software [7] provided by the Department of Empirical Inference for Machine Learning of the Max Planck Institute for biological cybernetics.

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